Cambright Solved Paper

Tags	2023 Additional Math CIE IGCSE May/June P2 V1
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Status	Done

1 Variables x and y are such that when $\lg y$ is plotted against $\sqrt{x}$ a straight line passing through the points (1, 5) and (2.5, 8) is obtained. Show that $y = A \times b^{\sqrt{x}}$ where A and b are constants to be found. [4]

Let's find the gradient of the line first

$$\text{Gradient} = \frac{8-5}{2.5-1} = \frac{3}{1.5} = 2$$

Now, using the straight line equation,

$$Y = mX + c$$

$$\lg y = 2\sqrt{x} + c$$

To find y-intercept or c, sub $\sqrt{x} = 1 \ and \ \lg y = 5$

$$5 = 2 \times 1 + c$$

$$c = 3$$

$$\lg y = 2\sqrt{x} + 3$$

$y = A \times b^{\sqrt{x}}$ can be written as

$$\lg y = \lg(A \times b^{\sqrt{x}})$$

$$\lg y = \lg A + \lg b^{\sqrt{x}}$$

$$\lg y = \sqrt{x} \lg b + \lg A$$

Here, we can easily see $\lg b = 2$ and $\lg A = 3$

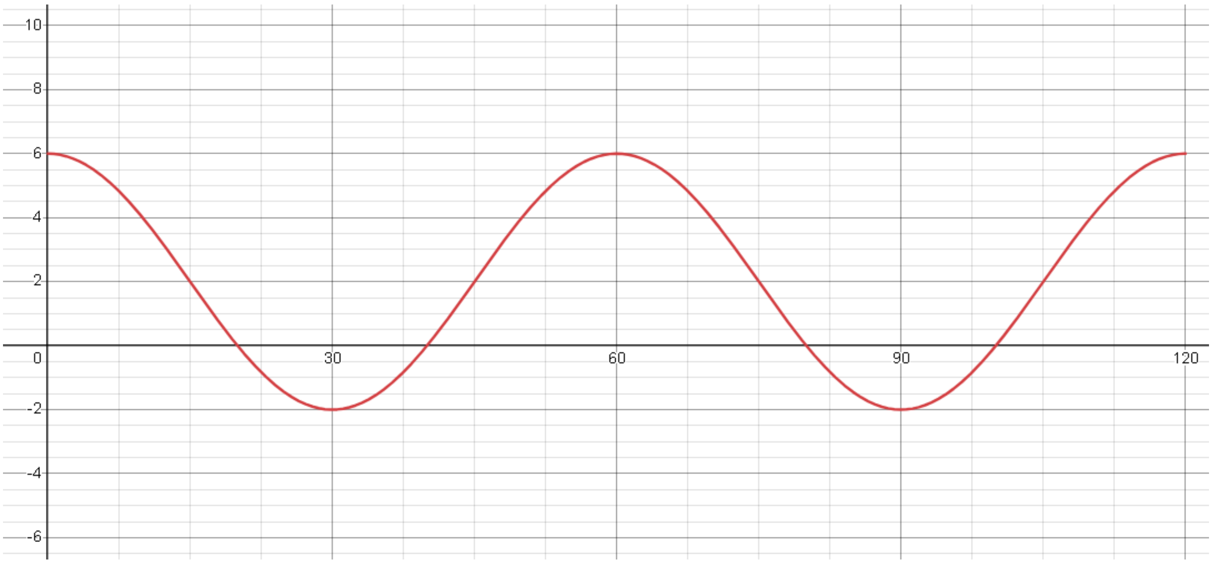
$b = 10^2 = 100$ and $A = 10^3 = 1000$

$$y = 1000 \times 100^{\sqrt{x}}$$

2 The function g is defined for $0° \leqslant x \leqslant 120°$ by $g(x) = 2 + 4\cos 6x$.

(a) On the axes, sketch the graph of $y = g(x)$. [3]

Make sure your calculator is set in degrees mode before drawing anything



https://www.desmos.com/calculator/k1iuqcsfur

(b) State the amplitude of g. [1]

(c) State the period of g. [1]

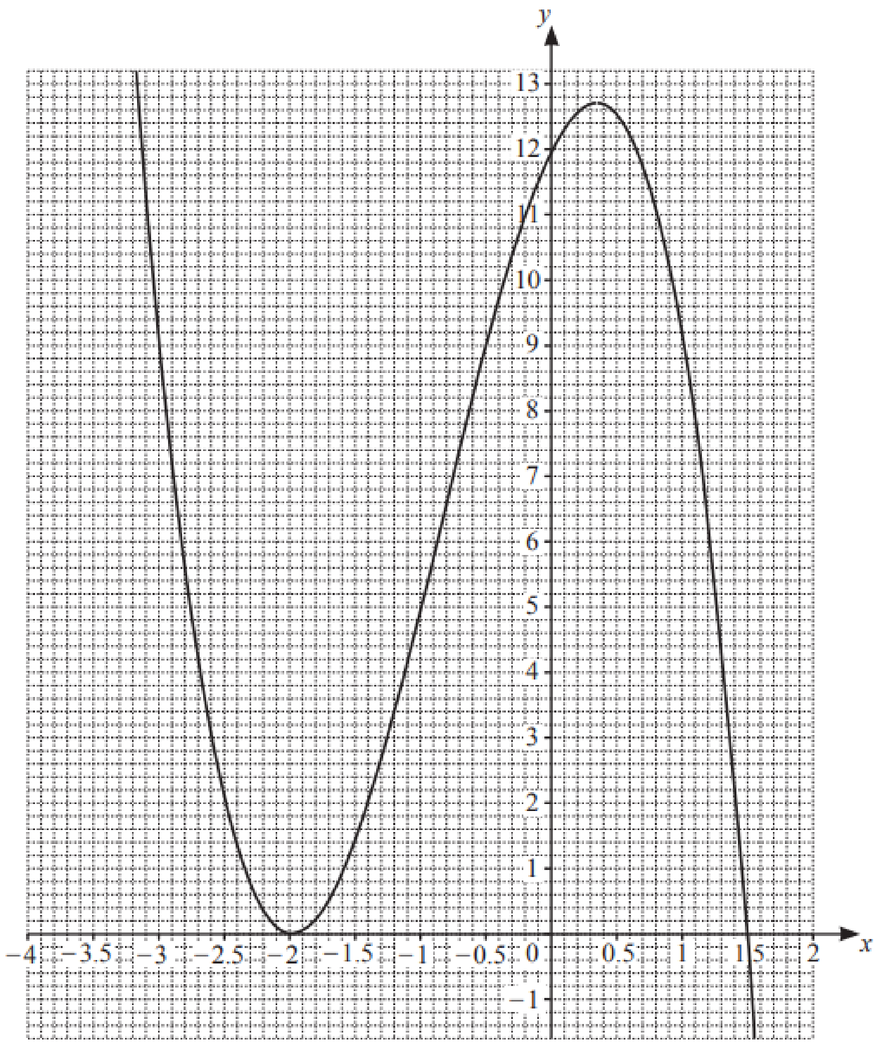
In $y = a\cos bx + c,\ a = \text{amplitude},\ \dfrac{360^\circ}{b} = \text{period},\ and\ c = \text{vertical shift}$

$\text{(b)amplitude} = 4$

$\text{(c)} = \dfrac{360^\circ}{b} = 6$ (if you extend the graph till 360, there will be 6 cos waves)

$b = \dfrac{360^\circ}{6} = 60^\circ$

3



The diagram shows the graph of $y = \text{h}(x)$ where $\text{h}(x) = (x+a)^2(b+cx)$ and a, b and c are integers. The curve meets the x-axis at the points $(-2, 0)$ and $(1.5, 0)$ and the y-axis at the point $(0, 12)$.

(a) Find the values of a, b and c. [2]

In any function $f(x) = (x+a)^2(x+b)$, $-a$ will always be the point on the graph which is both an x-intercept and stationary point.

Since -2 is both an x-intercept and stationary point, $-a = -2 \rightarrow a = 2$

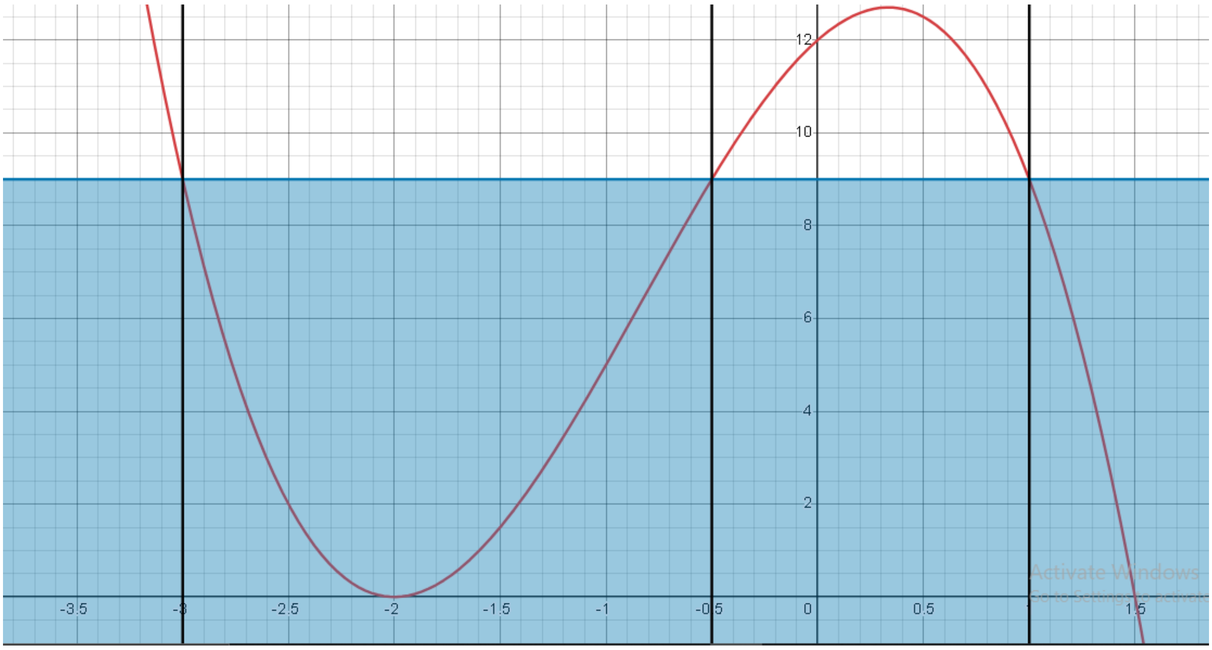
$(b + cx)$ leads to an x-intercept of 1.5, meaning when $(b + cx) = 0$, it is 1.5

$b + 1.5c = 0$

If b = 3 and c = -2, they cancel out $3 + (-2 \times 1.5) = 0$

$a = 2,\ b = 3,\ c = -2$

(b) Use the graph to solve the inequality $h(x) \leqslant 9$. [3]



https://www.desmos.com/calculator/f0eplqltwi

Using the graph, we see that when x is greater than -3 and smaller than -0.5, and x is greater than 1, we get our values

$$-3 \leq x \leq -0.5 \; and \; x \geq 1$$

4 **(a)** Solve the equation $5^{2y-1} = 6 \times 3^y$, giving your answer correct to 3 decimal places. [3]

Take the log of both sides

$$\log 5^{2y-1} = \log(6 \times 3^y)$$

$$(2y-1)\log 5 = \log 6 + y \log 3$$

$$2y \log 5 - \log 5 = \log 6 + y \log 3$$

Collect like terms

$$2y \log 5 - y \log 3 = \log 6 + \log 5$$

$$y(2\log 5 - \log 3) = \log(6 \times 5)$$

$$y(\log \frac{5^2}{3}) = \log 30$$

$$y(\log \frac{25}{3}) = \log 30$$

$$y = 1.604$$

(b) Solve the equation $e^{2x} - 4 + 3e^{-2x} = 0$, giving your answers in exact form. [4]

Multiply both sides by e^{2x}

$e^{4x} - 4e^{2x} + 3 = 0$

Let $u = e^{2x}$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$u = 3 \text{ or } u = 1$

$e^{2x} = 3 \text{ or } e^{2x} = 1$

$2x = \ln 3 \text{ or } x = 0$

$x = \frac{1}{2}\ln 3 \text{ or } x = 0$

5 The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume of a sphere is increasing at a constant rate of $24\,\text{cm}^3\,\text{s}^{-1}$. Find the rate of increase of the radius when the radius is 6 cm. [4]

$$\frac{dV}{dr} = 4\pi r^2$$

$$r = 6 \rightarrow \frac{dV}{dr} = 4\pi 6^2 \rightarrow \frac{dV}{dr} = 144\pi$$

$$\frac{dV}{dt} = 24cm^3s^{-1},\ \frac{dr}{dV} = \frac{1}{144\pi}$$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$\frac{dr}{dt} = \frac{1}{144\pi} \times 24$$

$$\frac{dr}{dt} = 0.0531$$

6 **(a)** The position vectors of the points P, Q and R relative to an origin O are $\begin{pmatrix}4\\7\end{pmatrix}$, $\begin{pmatrix}8\\5\end{pmatrix}$ and $\begin{pmatrix}x\\y\end{pmatrix}$ respectively. The point R lies on PQ extended such that $3\overrightarrow{QR} = 2\overrightarrow{PR}$. Use a vector method to find the values of x and y. [3]

$$3\overrightarrow{QR} = 3\begin{pmatrix}x-8\\y-5\end{pmatrix}$$

$$2\overrightarrow{PR} = 2\begin{pmatrix}x-4\\y-7\end{pmatrix}$$

$$3\begin{pmatrix}x-8\\y-5\end{pmatrix} = 2\begin{pmatrix}x-4\\y-7\end{pmatrix}$$

Solving for x

$$3(x-8) = 2(x-4)$$

$$3x - 24 = 2x - 8$$

$$x = 16$$

Solving for y

$$3(y-5) = 2(y-7)$$

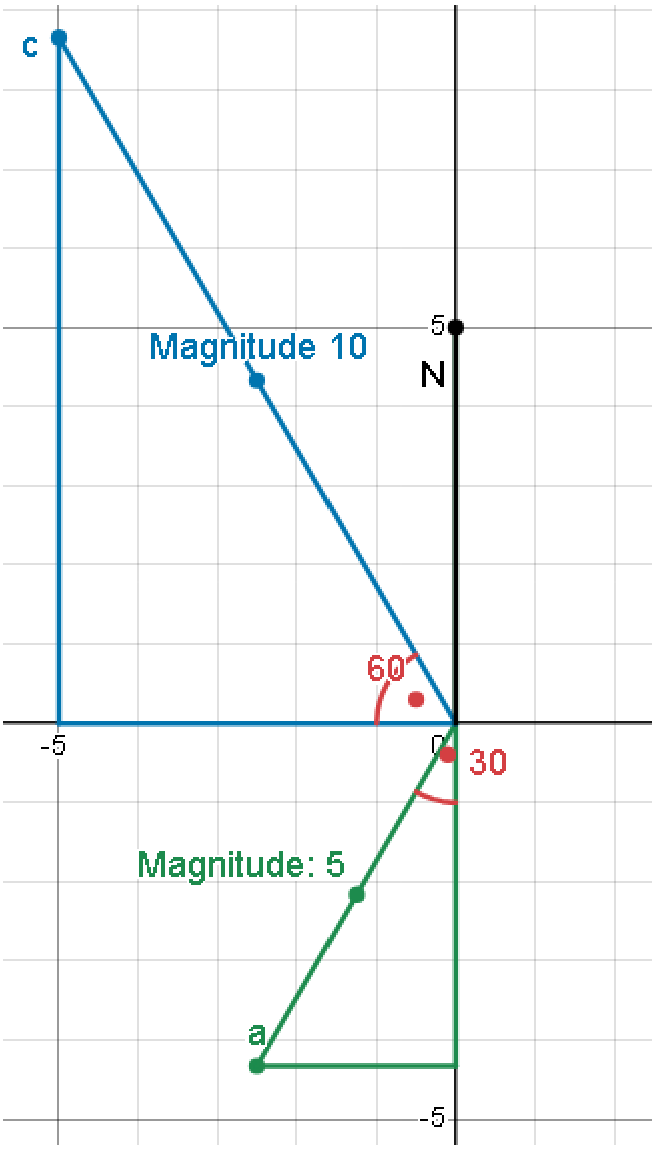
$$3y - 15 = 2y - 14$$

$$y = 1$$

(b) You are given that **i** is a unit vector due east and **j** is a unit vector due north.

Three vectors, **a**, **b** and **c** are in the same horizontal plane as **i** and **j** and are such that **a** + **b** = **c**.
The magnitude and bearing of **a** are 5 and 210°.
The magnitude and bearing of **c** are 10 and 330°.

(i) Find **a** and **c** in terms of **i** and **j**. [2]



https://www.desmos.com/calculator/fbwlqi44n9

As you can see we get 2 triangles if we draw a rough sketch. The triangle of a has an angle of 30° and the triangle of c has an angle of 60°.

Don't forget to change your calculator to degrees!

In triangle a, $\sin 30^\circ = \frac{x}{5}$ (opposite over hypotenuse)

$x = 5\sin 30^\circ$

$x = 2.5,\ \text{since x is going in the negative direction, } x = -2.5$

$\cos 30^\circ = \frac{y}{5}$ (adjacent over hypotenuse)

$y = 5\cos 30^\circ$

$y = \frac{5\sqrt{3}}{2},\ \text{since y is going in the negative direction, } y = -\frac{5\sqrt{3}}{2}$

$\therefore a = -2.5i - \frac{5\sqrt{3}}{2}j$

In triangle c, $\sin 60^\circ = \frac{y}{10}$

$y = 10\sin 60^\circ$

$y = 5\sqrt{3}$

$\cos 60^\circ = \frac{x}{10}$

$x = 10\cos 60^\circ$

$x = 5,\ \text{since x is going in the negative direction, } x = -5$

$\therefore c = -5i + 5\sqrt{3}j$

(ii) Find the magnitude and bearing of **b**. [5]

Since $a + b = c,\ b = c - a$

$b = (-5i + 5\sqrt{3}j) - (-2.5i - \frac{5\sqrt{3}}{2}j)$

Collect like terms

$(-5i) - (-2.5i) = -5i + 2.5i = -2.5i$

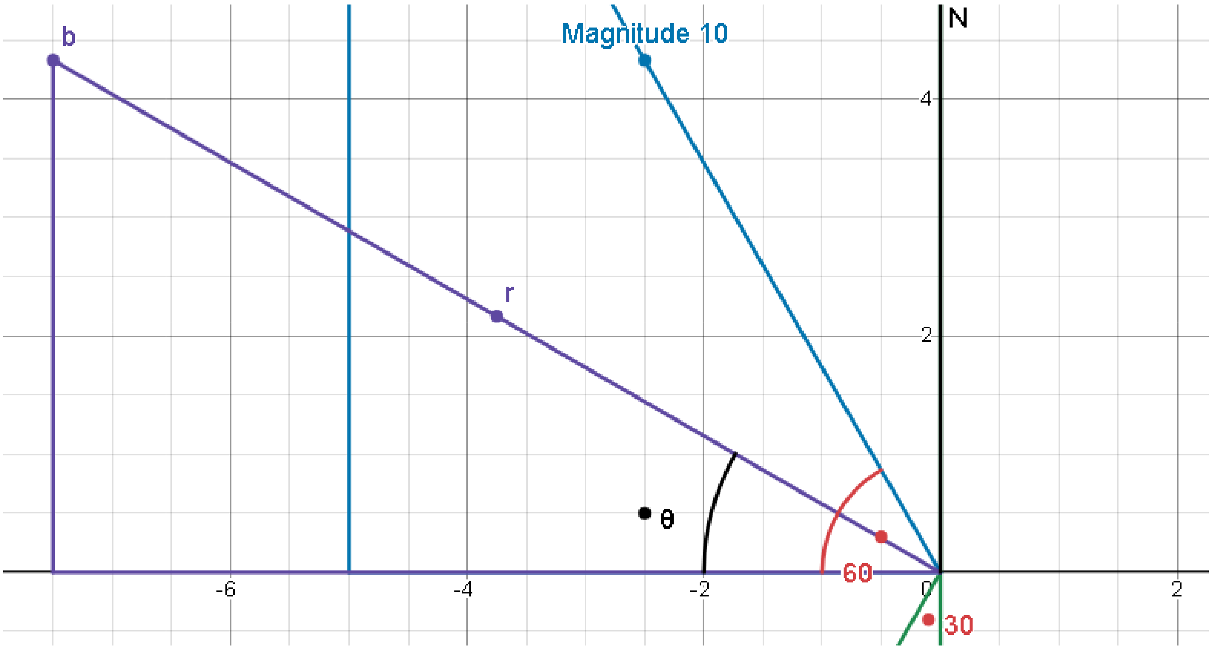
$(5\sqrt{3}j) - (-\frac{5\sqrt{3}}{2}j) = 5\sqrt{3}j + \frac{5\sqrt{3}}{2}j = \frac{15\sqrt{3}}{2}j$

$\therefore b = -2.5i + \frac{15\sqrt{3}}{2}j$

$$Magnitude\ r = \sqrt{(-2.5)^2 + (\frac{15\sqrt{3}}{2})^2}$$

$$Magnitude\ r = \sqrt{6.25 + 168.75}$$

$$r = 13.2$$



https://www.desmos.com/calculator/fbwlqi44n9 PS: this is not a 100% accurate image, it is simply used for visual purposes and comprehension.

Since we've found both x and y, we need to find the angle θ

$$\tan\theta = \frac{y}{x}$$

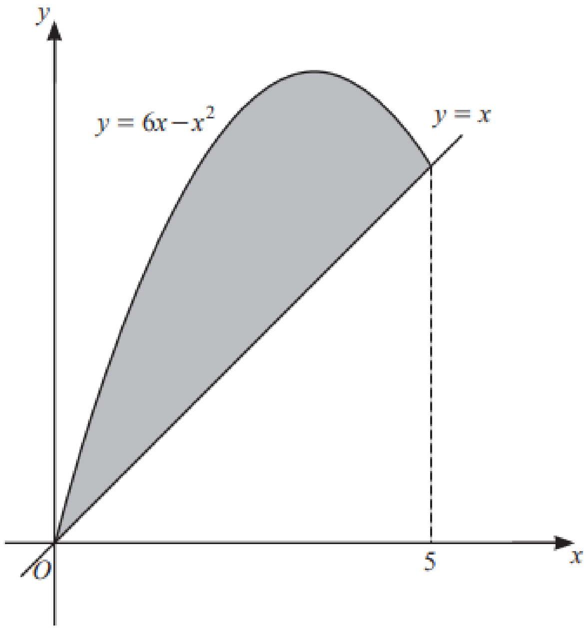
$$\tan\theta = \frac{\frac{15\sqrt{3}}{2}}{-2.5}$$

$$\theta = 79.1^\circ$$

Since this angle θ was found from above the y-axis, to get the bearing we add

$$270^\circ + 79.1^\circ = 349.1^\circ$$

7 (a)



The diagram shows the curve $y = 6x - x^2$ for $0 \leqslant x \leqslant 5$ and the line $y = x$. Find the area of the shaded region. [4]

Shaded area $=$ Curve area $-$ Line area

Curve area $= \int_0^5 6x - x^2 dx$

$= [\frac{6x^2}{2} - \frac{x^3}{3}]_0^5$

$= [3x^2 - \frac{x^3}{3}]_0^5$

$= [3(5)^2 - \frac{5^3}{3}] - 0$

$= 75 - \frac{125}{3}$

Line area $= \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

Shaded area $= 75 - \frac{125}{3} - \frac{25}{2} = \frac{125}{6}$

(b) **(i)** Find $\int \left(\frac{1}{(2x-6)^3} + \cos x \right) \mathrm{d}x$. [3]

$$\int \frac{1}{(2x-6)^3} dx + \int \cos x \ dx$$

Let $2x - 6 = u$, $\frac{du}{dx} = 2, \ dx = \frac{du}{2}$

$$\int \frac{1}{u^3} \frac{du}{2} + \int \cos x \ dx$$

$$\frac{1}{2} \int u^{-3} du + \sin x + c$$

$$\frac{1}{2} \times \frac{u^{-2}}{-2} + \sin x + c$$

$$\frac{(2x-6)^{-2}}{-4} + \sin x + c$$

(ii) Find $\int \frac{(x^4+1)^2}{2x} \mathrm{d}x$. [3]

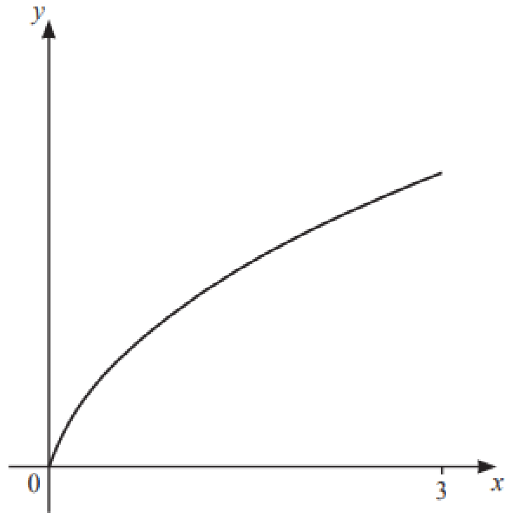
$$\int \frac{x^8+2x^4+1}{2x} dx$$

$$\int \frac{x^7}{2} + x^3 + \frac{1}{2x} dx$$

$$\frac{x^8}{2 \times 8} + \frac{x^4}{4} + \frac{1}{2} \ln x + c$$

$$\frac{x^8}{16} + \frac{x^4}{4} + \frac{1}{2} \ln x + c$$

8 **(a)**



The diagram shows the graph of $y = f(x)$ where f is defined by $f(x) = \dfrac{3x}{\sqrt{5x+1}}$ for $0 \leqslant x \leqslant 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1}. [3]

Range of $f(x) : x = 0 \rightarrow f(x) = 0,\ x = 3 \rightarrow f(x) = 2.25$

Domain of $f(x) =$ Range of $f^{-1}(x)$

Range of $f(x) =$ Domain of $f^{-1}(x)$

$\therefore$ Domain of $f^{-1}(x) : 0 \leq x \leq 2.25$

Range of $f^{-1}(x) : 0 \leq f^{-1} \leq 3$

(ii) Solve the equation $f(x) = x$. [2]

$$\frac{3x}{\sqrt{5x+1}} = x$$

$$3x = x\sqrt{5x+1}$$

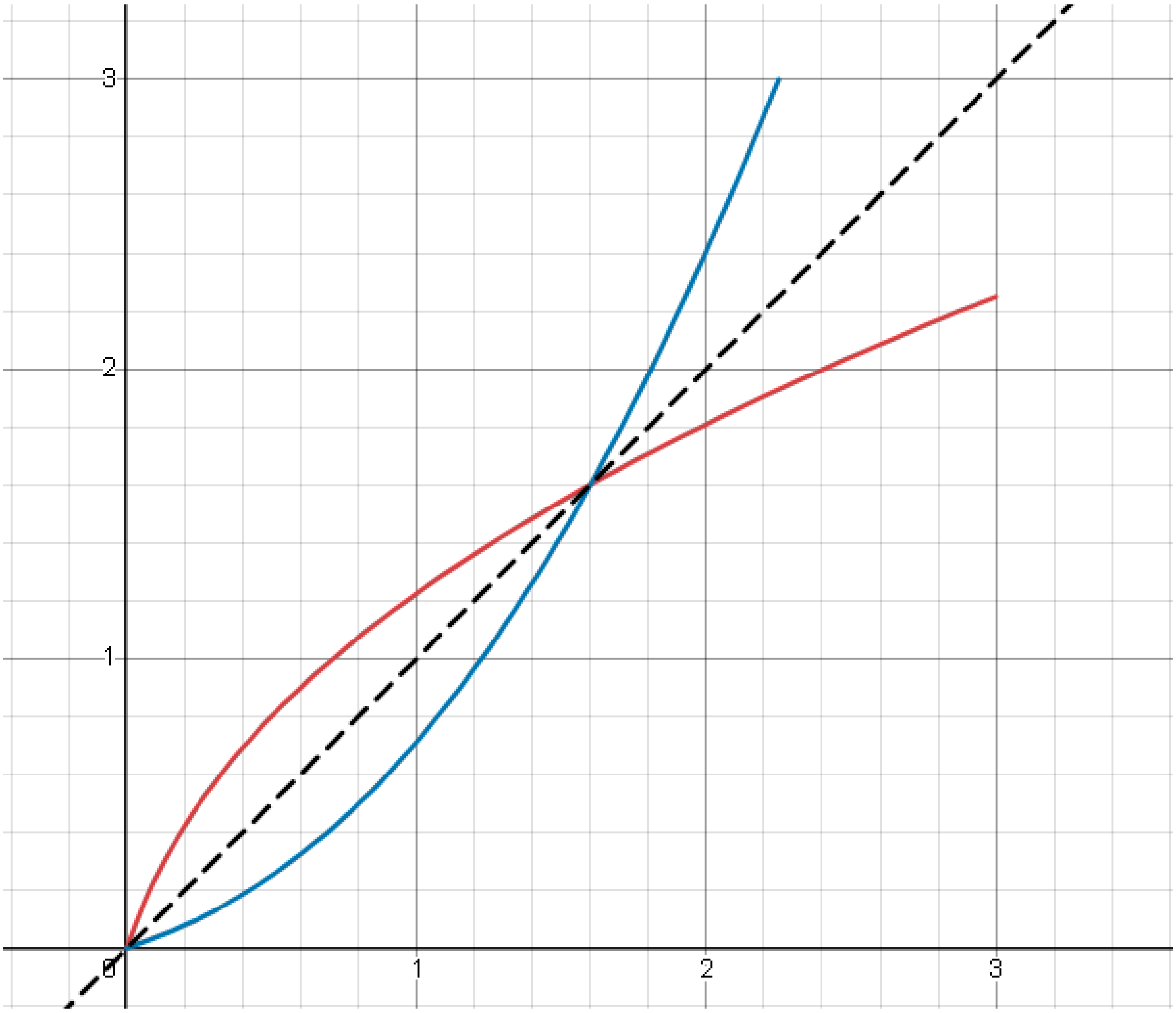
$$3 = \sqrt{5x+1} \ or \ x = 0$$

$$5x + 1 = 9$$

$$5x = 8$$

$$x = \frac{8}{5} \ or \ x = 0$$

(iii) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]



https://www.desmos.com/calculator/uahxh4fdyp

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for} \quad x \geqslant 1,$$

$$h(x) = e^{4x} \quad \text{for} \quad x \geqslant k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

$$y = \sqrt[3]{8x^3 + 3}$$

$$x = \sqrt[3]{8y^3 + 3}$$

$8y^3 + 3 = x^3$

$8y^3 = x^3 - 3$

$y^3 = \frac{x^3-3}{8}$

$y = \sqrt[3]{\frac{x^3-3}{8}}$

$g^{-1}(x) = \sqrt[3]{\frac{x^3-3}{8}}$

(ii) State the least value of the constant k such that gh(x) can be formed. [1]

(iii) Find and simplify an expression for gh(x). [1]

(ii) When e has an exponent of less than 0, the value is less than 1. Since we want $x \geq 1$ for $g(x)$,

$k = 0$ is the smallest value possible.

(iii)

$gh(x) = g(e^{4x})$

$= \sqrt[3]{8(e^{4x})^3 + 3}$

$= \sqrt[3]{8e^{12x} + 3}$

9 In this question all lengths are in centimetres and all angles are in radians.

(a) The area of a sector of a circle of radius 24 is 432 cm^2. Find the length of the arc of the sector. [4]

Don't forget, calculator in radians mode!

To find the arc length $r\theta$, we know the radius is 24 but not the angle θ

$\text{Sector area} = \frac{1}{2}r^2\theta$

$432 = \frac{1}{2}24^2\theta$

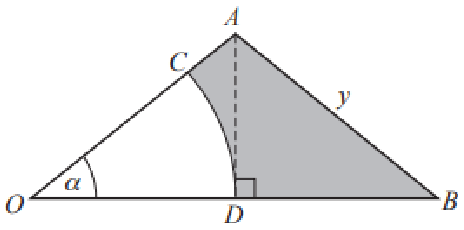
$864 = 576\theta$

$\theta = 1.5\ rad$

$\text{Arc length} = r\theta$

$24 \times 1.5 = 36\ cm$

(b)



The diagram shows an isosceles triangle, OAB, with $AO = AB = y$ and height AD. OCD is a sector of the circle with centre O. Angle AOB is α.

(i) Find an expression for OB in terms of y and α. [1]

In triangle AOD, $\cos\alpha = \dfrac{OD}{y}$, since AO = y

$OD = y\cos\alpha$

Since, $OD = DB,\ OB = 2y\cos\alpha$

(ii) Hence show that the area of the shaded region can be written as $\frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$. [3]

Shaded area = triangle area - sector area

$\text{Triangle area} = \dfrac{1}{2}bh = \dfrac{1}{2}2y\cos\alpha \times AD$

$\text{Triangle area} = \dfrac{1}{2}2y\cos\alpha \times y\sin\alpha\ (\sin\alpha = \dfrac{AD}{y})$

$\text{Triangle area} = \dfrac{y^2}{2}2\cos\alpha\sin\alpha$

$\text{Sector area} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(y\cos\alpha)^2\alpha = \dfrac{y^2}{2}\cos^2\alpha \times \alpha$

Shaded area $= \frac{y^2}{2} 2\cos\alpha\sin\alpha - \frac{y^2}{2}\cos^2\alpha \times \alpha$

Factor out $\frac{y^2}{2}\cos\alpha$

Shaded area $= \frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$

10 In the expansion of $\left(ax + \frac{b}{x^2}\right)^9$, where a and b are constants with $a > 0$, the term independent of x is $-145\,152$ and the coefficient of x^6 is -6912. Show that $a^2b = -12$ and find the value of a and the value of b. [7]

Term independent of x happens when $(ax)^6 \times (\frac{b}{x^2})^3$, this will happen at ${}^9C_3$

Term independent of x: ${}^9C_3 \times (ax)^6 \times (\frac{b}{x^2})^3 = 84a^6b^3$

$84a^6b^3 = -145,152$

$a^6b^3 = -1728$

$(a^2b)^3 = (\sqrt[3]{-1728})^3$

$a^2b = -12$

$b = -\frac{12}{a^2}$

Coefficient of x^6 happens when $(ax)^8 \times (\frac{b}{x^2})^1$, this will happen at ${}^9C_1$

Coefficient of x^6 : ${}^9C_1 \times (ax)^8 \times (\frac{b}{x^2})^1 = 9a^8b^2$

$9a^8b = -6912$

$a^8b = -768$

$a^8 \times -\frac{12}{a^2} = -768$

$a^6 = 64$

$a = 2$

$b = -\frac{12}{2^2} = -\frac{12}{4} = -3$

$a = 2,\ b = -3$

11 The line with equation $x+3y=k$, where k is a positive constant, is a tangent to the curve with equation $x^2+y^2+2y-9=0$. Find the value of k and hence find the coordinates of the point where the line touches the curve. [9]

$x = k - 3y$

Sub this value in the curve equation

$(k-3y)^2 + y^2 + 2y - 9 = 0$

$k^2 - 6ky + 9y^2 + y^2 + 2y - 9 = 0$

Collect like terms

$10y^2 + (2-6k)y + k^2 - 9 = 0$

Using the discriminant, we want to find one root only since it is a tangent

$b^2 - 4ac = 0$

$(2-6k)^2 - 4(10)(k^2-9) = 0$

$4 - 24k + 36k^2 - 40k^2 + 360 = 0$

$-4k^2 - 24k + 364 = 0$

$k^2 + 6k - 91 = 0$

$(k-7)(k+13) = 0$

$k = 7 \; or \; k = -13$

Since k is a positive constant,

$k = 7$

To find the coordinates,

$10y^2 + (2-6k)y + k^2 - 9 = 0$

$10y^2 + (2 - 6 \times 7)y + 7^2 - 9 = 0$

$10y^2 + (-40)y + 40 = 0$

$y^2 - 4y + 4 = 0$

$(y-2)(y-2) = 0$

$y = 2$

Sub y = 2 in the line equation

$x = 7 - 3(2)$

$x = 7 - 6 \rightarrow x = 1$

Additional notes

Websites and resources used:

- Desmos graphing calculator

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.